

Assignment Quiz 5
November 7, 1997

Instructor: B.L. Daku
Time: 15 minutes
Note: No aids

Name:
Student Number:

2

1. For the following Z-transforms, what are the defining characteristics of the inverse Z-transform. Use only the characteristics from the following list.

- finite duration
- infinite duration
- causal
- anticausal
- two-sided

(a) $X_1(z)$; ROC Entire Z-plane except $z = \infty$.

causal

(b) $X_2(z)$; ROC $|z| > r_2$.

infinite duration

(c) $X_3(z)$; ROC $r_2 < |z| < r_1$.

finite duration

2. Determine the Z-transform of the sequence $x(n]$ (remember the ROC).

$$x(n) = -r^n e^{j\theta n} u(n-1)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{-1} (-r^n e^{j\theta n}) z^{-n} = \sum_{n=-\infty}^{-1} (-re^{j\theta})^n z^{-n}$$

$$= \frac{1}{1 - (-re^{j\theta})z^{-1}}$$

$$X(z) = \frac{1}{1 + re^{j\theta} z^{-1}}$$

ROC: $0 < 1 + re^{j\theta} z^{-1}$
 $-re^{j\theta} z^{-1} < 1$
 $-re^{j\theta} < z$

$$\boxed{|re^{j\theta}| < |z|}$$

Instructor: B.L. Daku
Time: 15 minutes
Note: No aids

Name:
Student Number:

1. Find the inverse z-transform of

$$X(z) = \frac{2z^2 - 3.25z}{(z - 0.25)(z - 3)} \quad \text{ROC } 0.25 < |z| < 3.$$

$$\therefore \frac{X(z)}{z} = \frac{2z - 3.25}{(z - 0.25)(z - 3)}$$

$$\therefore \frac{X(z)}{z} = \frac{A}{(z - 0.25)} + \frac{B}{(z - 3)}$$

$$\frac{X(z)}{z} = \frac{1}{(z - 0.25)} + \frac{1}{(z - 3)}$$

$$\therefore X(n) = (2^n) u(n) + (3^n) u(n)$$

$$A = \frac{2z - 3.25}{(z - 3)} \Big|_{z=0.25}$$

$$= \frac{0.5 - 3.25}{-2.75} = 1$$

$$B = \frac{2z - 3.25}{(z - 0.25)} \Big|_{z=3}$$

$$= \frac{6 - 3.25}{2.75} = 1$$

2. A system has an input $x(n) = \delta(n) + 3\delta(n-2)$ and an output $y(n) = a^n u(n) + 3a^{(n-2)} u(n-2)$. What is the system function and the impulse response of the system? What values of a will ensure that the system is stable?

$$X(z) = \frac{1}{z} + \frac{3}{z^2}$$

$$= \frac{z^{-1} + 3z^{-2}}{z^{-1}}$$

$$= 1 + 3z^{-1}$$

$$Y(z) = \frac{1}{1 - az^{-1}} + \frac{3}{1 - az^{-2}}$$

$$= \frac{1}{1 - az^{-1}} + \frac{3}{(1 - az^{-1})(1 - az^{-1})}$$

$$= \frac{1 + 3(1 - az^{-1})}{(1 - az^{-1})^2}$$

$$= \frac{4 - az^{-1}}{(1 - az^{-1})^2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4 - a(z^{-1} + 3z^{-2})}{(1 - az^{-1})(1 - az^{-2})}$$

$$\boxed{|a| < |z|}$$